3.4/3.5 Notes: Real and Complex Zeros

The possible rational zeros of

$$P(x) = \frac{\text{factors of constant term}}{\text{factors of leading coefficient}}$$

Reminder: the zeros of a function are the same as the x-intercepts of its graph.

3.4/3.5 Notes: Real and Complex Zeros

Factor and find the zeros:

a) $F(x) = x^4 + 8x^2 - 9$

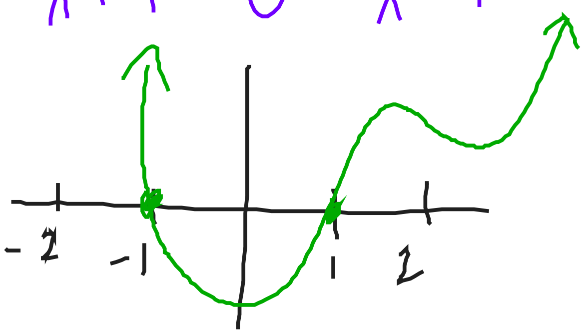
$$= (x^2 - 1)(x^2 + 9)$$

$$= (x+1)(x-1)(x^2 + 9)$$

zeros: $x = -1 \quad x = 1 \quad x = \pm 3i$

$x + 1 = 0 \quad x - 1 = 0 \quad x^2 + 9 = 0$

$\sqrt{x^2} = \sqrt{-9}$

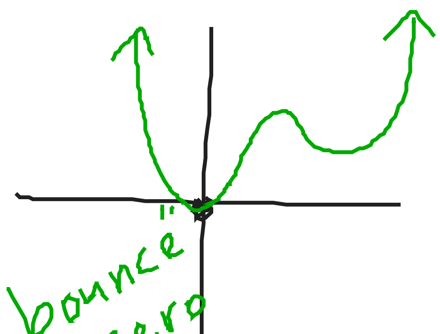


b) $P(x) = x^4 + 9x^2$

$$= x^2(x^2 + 9)$$

zeros: $x = 0$ multiplicity = 2

$x = \pm 3i$



"bounce" at zero

Factor and find the zeros:

15. $P(x) = 1x^3 + 2x^2 - 13x + 10$

possible zeros

$\pm 1 \quad \pm 2 \quad \pm 5 \quad \pm 10$

-1	1	2	-13	10
		-1	-1	14
	1	1	-14	$\boxed{24}$

1	1	2	-13	10
		1	3	-10
	1	3	-10	$\boxed{0}$

add

$(x-1)(x^2+3x-10)$

$\boxed{P(x) = (x-1)(x+5)(x-2)}$

Zeros
$x = 1$
$x = -5$
$x = 2$

Factor and find the zeros:

39. $P(x) = 2x^4 - 7x^3 + 3x^2 + 8x - 4$

yes

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -7 & 3 & 8 & -4 \\ & \downarrow & 1 & -3 & 0 & 4 \\ \hline & 2 & -6 & 0 & 8 & \boxed{0} \end{array}$$

$$\begin{array}{r|rrrr} 1 & 2 & -6 & 0 & 8 \\ & \downarrow & 2 & -4 & -4 \\ \hline & 2 & -4 & -4 & \boxed{4} \end{array}$$

no

$$\begin{array}{r|rrrr} -1 & 2 & -6 & 0 & 8 \\ & \downarrow & -2 & 8 & -8 \\ \hline & 2 & -8 & 8 & \boxed{0} \end{array}$$

$= \left(x - \frac{1}{2}\right)\left(2x^3 - 6x^2 + 8\right)$

$= \left(x - \frac{1}{2}\right)(x+1)\left(2x^2 - 8x + 8\right)$

$= \left(x - \frac{1}{2}\right)(x+1)\,2\left(x^2 - 4x + 4\right)$

$\boxed{= 2\left(x - \frac{1}{2}\right)(x+1)(x-2)^2}$

$\boxed{x = \frac{1}{2} \quad x = -1 \quad x = 2}$

3.4 CHECK EVENS

16. $P(x) = (x+2)(x+1)(x-7)$.

Therefore, the zeros are -2, -1, and 7.

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18. $P(x) = x^3 - 3x - 2 = (x-2)(x+1)^2$

Therefore, the zeros are 2 and -1.

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32. $P(x) = 2\left(x - \frac{1}{2}\right)(x+2)^2$.

Therefore, the zeros are -2 and $\frac{1}{2}$.